Mathematics

Prework Training

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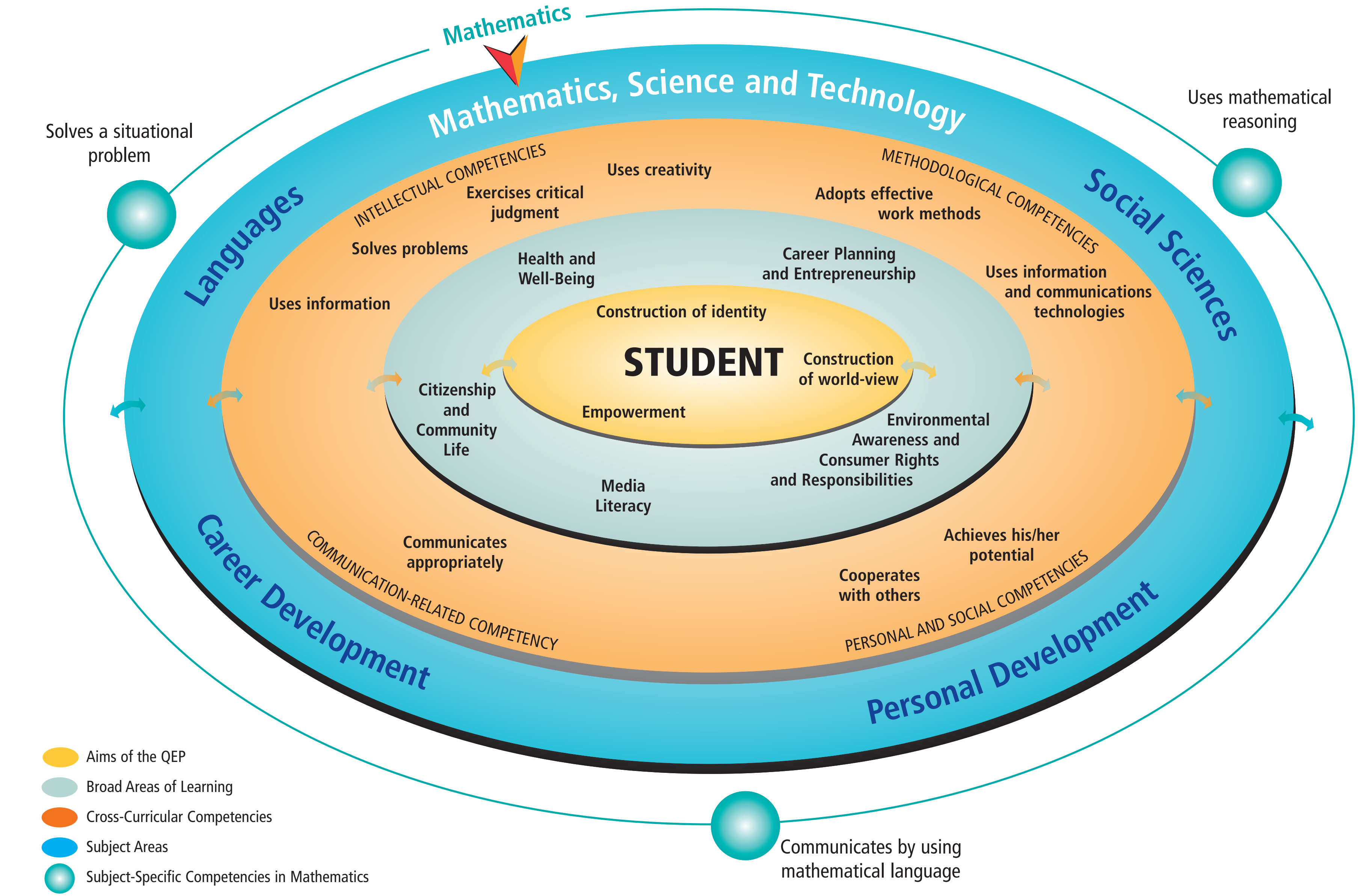

Work-Oriented Training Path, Prework Training

Introduction to the Program

Mathematics is a vast adventure; its history reflects some of the noblest thoughts of countless generations.
Dirk J. Struik

Mathematics helps us understand reality. It is used in a multitude of everyday activities at school, at home and at work. Our world is filled with symbols, numbers and statistics. Whether they are trying to understand how to put together a piece of furniture, adapting a recipe or calculating the price-quality ratio of a potential purchase, students are constantly faced with situations that require an understanding of mathematics.

The development of mathematical concepts is essential to the sociovocational integration of students in Prework Training. However, as a language and tool of abstraction, mathematics is particularly difficult for many of these students, since it deals with relationships between objects or between elements of a situation in abstract terms.

Because of the wide variety of learning students have acquired, this program is particularly difficult in terms of differentiation. Teachers must take each student's particular needs into account when choosing learning content. The idea is not to restrict the range of the different branches of mathematics, but rather to focus on selected elements of the discipline in accordance with each student's level of learning and, in some cases, job requirements or the possibility of going on to a program in a semiskilled trade. Teachers should also focus on concrete objects, make direct connections with practical applications and have students regularly apply their learning to other subjects in order to impress upon them the usefulness of mathematics. Finally, they must help students realize the extent to which they need mathematics in their everyday lives.

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By the end of the program, students will have exercised their mathematical competencies at home, in the workplace or in their recreational activities.

By the end of the program, students will have exercised their mathematical competencies at home, in the workplace or in their recreational activities. For example, they will need to have learned to solve transportation problems involving space, distance, time, cost, etc. During practicums, they will have applied and consolidated competencies that should enable them to hold a job. For example, a student who has learned to calculate correctly should be able to make change as a service station attendant, while a student who has developed spatial perception should be able to figure out how to display items in a store working as an assistant clerk. In their recreational activities, students will be encouraged to base their decisions on a cost analysis, taking into account their budget, the availability of services (e.g. location, distance) and potential health advantages (e.g. body mass, heart rate). In addition to these practical concerns, teachers should also help students as much as possible to develop logic, abstraction skills and the ability to use appropriate mathematical language when needed. Mathematics also helps students structure their thoughts and enhances their intellectual development.

Students in this program should be able to pursue their learning in mathematics based on their elementary and Secondary Cycle One learning since the competencies are almost identical:

– Solves a situational problem

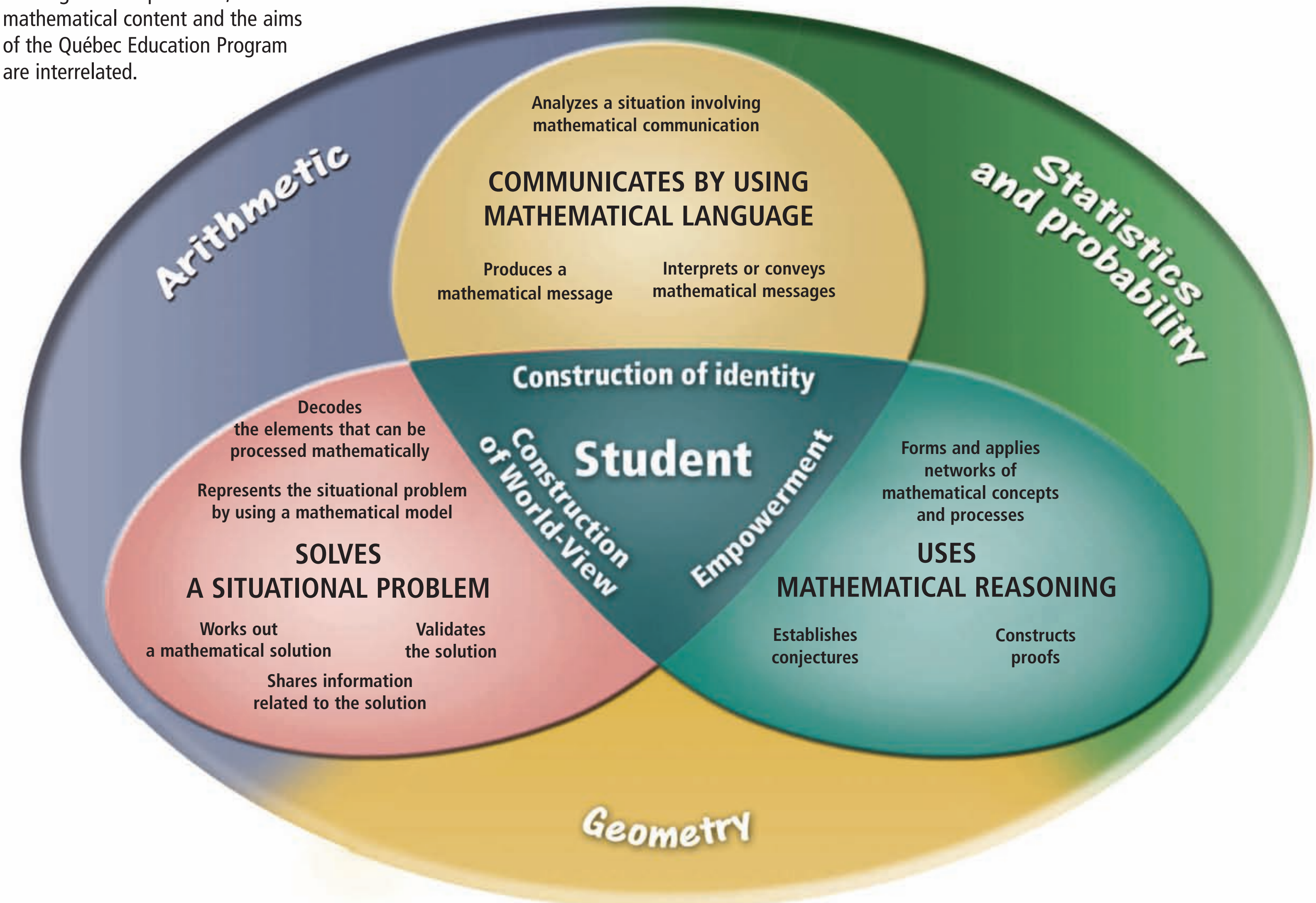
– Uses mathematical reasoning

– Communicates by using mathematical language

These competencies mirror three facets of mathematical activity essential to mathematical thought. Although teachers might find it useful to distinguish among them, they are rarely isolated in solving more or less complex situational problems. Teachers should combine them with the program content, which is divided into three branches of mathematics: arithmetic, statistics and probability, and geometry.

CONTRIBUTION OF THE MATHEMATICS PROGRAM TO THE STUDENT'S EDUCATION

The diagram below shows how the targeted competencies, the mathematical content and the aims of the Québec Education Program are interrelated.



Making Connections: Mathematics and the Other Subjects in Prework Training

Mathematics is present in every aspect of students' lives, whether they are satisfying their needs or managing their daily personal, family, work-related and recreational activities. Given the diversity of situations in which mathematics plays a role, many connections can be made with the other subjects.

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Learning in the subject area of language is essential to the development and application of mathematics competencies. The ability to read and assess various texts enables students to understand the elements of a situational problem. The ability to write various texts is necessary to explain the data associated with a problem and to present a structured solution. Finally, at every step of the process, students can apply their ability to communicate orally: by discussing their understanding of the situational problem, identifying missing or superfluous data, listening to others' opinions and assessing their relevance, and explaining their choice in a clear and structured manner.

In addition, the Autonomy and Social Participation course gives students a number of opportunities to apply their mathematical learning. The use of mathematical reasoning may prove useful in taking an informed position on everyday issues, for example, for interpreting interest rates, evaluating the cost of credit or comparing different ways of making a budget. Similarly, when they are asked to participate in the democratic life of their school, students can carry out a survey concerning a new rule or the popularity of a candidate to the student council, or organize data and represent results using graphs. The Physical Education and Health program provides other opportunities to use mathematics. For example, when developing their action plan to adopt a healthy, active lifestyle, students can calculate their body mass index and evaluate its impact on their health. The Technological and Scientific Experimentation program includes processes similar to the ones used in mathematics that make for enriching and stimulating connections. For example, students can be invited to use mathematical reasoning when observing a technical object or identifying how it works.

The diversity of learning and evaluation situations should enable students to make connections between mathematics and other subjects and to use mathematical language in everyday life.

Pedagogical Context

To learn to use their own intellectual resources, human beings must regularly frame and solve problems, make decisions, manage complex situations, and lead research and other types of projects, as well as processes with uncertain outcomes. If students are to construct their own competencies, these are the tasks they must be given, not occasionally, but every week, every day, in every sort of configuration.

Philippe Perrenoud

In order to develop mathematical competencies, students must be placed in learning and evaluation situations that foster rich and diverse learning. While referring to the appropriate branches of mathematics, these situations should ask questions such as "Why?," "Is this always true?" or "What would happen if . . .?"

The situational problems focus on obstacles to be overcome. They may take the form of exploration activities that allow students to conjecture,[1] simulate, experiment, develop arguments and draw conclusions. The use of problems that young adults or workers encounter in their everyday lives is especially important to kindle students' interest. Situations that focus on Competency 3 (Communicates by using mathematical language) may involve keeping a log, formulating an explanation, presenting a process, discussing or debating. Where the situations allow, establishing intradisciplinary and interdisciplinary links is particularly useful in helping students consolidate and transfer their learning.

The level of complexity of the tasks proposed should be appropriate to each student's level of competency development.

Differentiated instruction is absolutely essential to the students' success. The level of complexity of the tasks proposed should be appropriate to each student's level of competency development. If necessary, teachers can simplify a given task by modifying some of its parameters or by offering assistance. They may propose a task that is easier to define and carry out and limit the constraints of the situation. The situation may require little abstraction, involve the use of a limited number of mathematical concepts and processes, or include fewer steps. It may be familiar to the students, making the task easier, or it may allow for the use of technological tools such as geometry software, spreadsheet software, etc.

Hands-on activities are known to be important factors in the construction of mathematical concepts. Students use different resources depending on the activity. They may use various objects, blocks or geometry sets, graph or dotted paper, a calculator or software, or become familiar with instruments such as a stopwatch, an odometer and tools used in the workplace.

Teachers should use cultural references to demonstrate the importance of mathematics in everyday life and its role in history, as well as the contribution of mathematicians to the development of the discipline. They should select learning and evaluation situations that enable students to connect these references in a concrete way. The situations could include such things as historical accounts, research, interdisciplinary activities or the composition of a newspaper article. In addition, in developing learning and evaluation situations, the teacher should take into account students' need to make explicit connections between their mathematical knowledge and their employment training, in particular by using activities carried out in the workshop or in a practicum setting.

Each of the competencies in this program contributes to students' learning. Thus, as much as possible, it is preferable, whenever possible, to observe the full or partial development of these competencies in a given situation. However, when the time comes to evaluate the level of development of each

1. In this program the term *conjecture* indicates a statement that one suspects to be true. The verb *to conjecture* means to infer the truth of a statement and attempt to prove it. The subject of the conjecture (statement) may be mathematical (e.g. When I double the length of the side of a square, I quadruple its area). It may also be contextual (e.g. The most advantageous vacation package for the client is the one offered by company XYZ).

competency, it might be useful to use situational problems, application situations and communication situations that make it possible to evaluate each one separately.

The situational problems used to evaluate the competency *Solves a situational problem* require a new combination of previously learned concepts and processes. The complexity of a situational problem is characterized in particular by the scope of the knowledge mobilized and by the necessary connections between different branches of mathematics (arithmetic, probability and statistics, and geometry).

The application situations used to evaluate the competency *Uses mathematical* reasoning require a familiar combination of previously learned concepts and processes. They also require that students justify their actions or explain their reasoning, taking a position on their own or someone else's conjecture. The complexity of an application situation is characterized in particular by the quantity of concepts and processes mobilized.

Finally, the communication situations used to evaluate the competency *Communicates by using mathematical language* involve the production or interpretation of messages using different types of representation (e.g. words, symbols, drawings, grids) and previously learned concepts and processes. The complexity of these situations, addressed orally or in writing, is largely based on the transition from one type of representation to another in order to transmit information.

Example of a Learning and Evaluation Situation in Context

In an environment-related project focused on investigating a population's degree of awareness with respect to recycling, students must determine the quantity of waste produced and recycled.

Encouraged to research the topic, they mobilize and enrich their networks of concepts and processes. They analyze different information, consult resource people in their community and refer to on-line documents, newspapers or books containing information presented in different forms: texts, numerical data, percentages, tables and diagrams.

They interpret these data using their number sense and their understanding of proportionality. They establish a communication plan in which they use different types of representation, such as tables and graphs, to visualize the data and gather additional information. Some technological tools are particularly effective for this type of representation. For example, students could use the "insert table" function of Word, draw a diagram using an ideas manager or create a graph using spreadsheet software ("insert graph" function). This can lead them to make comparisons.

Then they produce a message taking into account the rules and conventions of everyday and mathematical language. Depending on whether they present the results of their research orally or in writing, they select the most appropriate means of communication: written report, poster, transparency, slide show or presentation followed by a discussion.

COMPETENCY 1 Solves a situational problem

An expert problem solver must be endowed with two incompatible qualities: a restless imagination and a patient pertinacity.
Howard W. Eves

Focus of the Competency

In mathematics, a situational problem involves a goal to be achieved and a task to be carried out, and students must be required to find a coherent solution to a problem. The problem should present a realistic challenge, stimulate students' interest and involvement, and encourage them to find a solution. To solve it, students must use a discovery approach, but it should also offer them an opportunity to reflect on that approach. Situational problems may address more or less familiar practical issues based on real-life situations, as well as purely mathematical questions.

A situational problem should represent a realistic challenge, stimulate students' interest and involvement, and encourage them to find a solution.

Solving a situational problem involves discernment, research and the development of strategies.[2] Students must define the situation, represent it mathematically, develop a solution, and validate and share it. This is a dynamic process that involves the ability to envisage possibilities, review one's work and exercise critical judgment. The ability to solve a situational problem is an effective intellectual tool that will help students develop other intellectual abilities that combine reasoning and creative intuition.

Students' development of this competency is based on their prior knowledge and their ability to solve situational problems in new contexts and to enrich their repertoire of strategies. Related to the broad areas of learning, the learning contexts should be based on their everyday or work-related lives. For example, they will learn to plan purchases by considering various possibilities, to compare athletes' performances using statistics, and to understand and carry out activities that require the use of geometry in everyday or work situations.

In previous years, the students recognized relevant information and identified implicit information in a situational problem. They decoded situational problems in which certain information was missing or which required a problem-solving process in several steps. They used different types of representation and strategies to develop a solution. They learned to validate their solution and convey it using mathematical language.

In Prework Training, the situational problems are slightly more complex and generally draw on the student's knowledge of several branches of mathematics, according to the particular problem. Solving a situational problem requires the use of a greater number of strategies.

The following examples illustrate the contribution of each branch of mathematics to the development of the competency.

- In arithmetic, the students use their number and operation sense as well as their understanding of the relationships between operations. Their comprehension of a situational problem should enable them to distinguish between explicit and implicit information on the one hand, and missing or unknown information on the other, and to illustrate relationships using various types of representation. They learn to use different strategies as they explore possible solutions. They apply the concept of proportionality to decision making and the choice of options for performing the tasks. Throughout the process, they work with, estimate, validate and interpret data and numerical expressions, in different forms of notation, taking their relative value into account depending on the context.

2. See examples of these strategies on pp. 30-33.

– In statistics and probability, the students use their understanding of information whose source may be observations, statistics or random experiments, to identify and process situational problems related to this branch. They use diagrams and tables to represent a situational problem, organize and analyze data and facilitate enumeration. They use the concepts of chance and random experiment to validate or refute certain predictions and perceptions current in society. They exchange information about the solution with their classmates, explaining their procedure, decisions, recommendations or conclusions. In their classmates' reactions, they identify means of evaluating the effectiveness of their solution or the reliability of their study. At every step of the problem-solving process, they may decide to use simulations when experiments pose a problem.

– In geometry, students who are solving situational problems use their spatial sense and understanding of measurement to identify the task and explore possible solutions. They create a mental representation of the figures involved in the situational problem. They represent two- and three-dimensional objects in different ways, using geometry sets or software as needed.[3]

When developing a solution involving finding unknown lengths, areas or volumes, they use definitions, properties or relationships by working with numerical expressions. They organize and justify the steps in their procedure using accepted properties and statements. They make sure that their result is plausible given the context and express it in the appropriate unit of measurement. They take advantage of discussions in which solutions are shared to enrich their network of relationships and strategies.

The competency *Solves a situational problem* has five key features: *Decodes the elements that can be processed mathematically*; *Represents the situational problem using a mathematical model*; *Works out a mathematical solution*; *Validates the solution*; and *Shares information related to the solution*.

3. For information on the appropriate mathematical tools, consult the Web site of the Service national du RECIT Mathématique, Science et Technologie: http://recitmst.qc.ca/.

Key Features of Competency 1

Solves a situational problem

Decodes the elements that can be processed mathematically

Derives information from various types of representations: linguistic, numerical, symbolic, graphic • If necessary, identifies any missing, additional or superfluous information • Identifies and describes the task to be performed by focusing on the question being asked or by formulating one or more questions

Represents the situational problem using a mathematical model

Associates a suitable mathematical model with the situational problem • If necessary, compares the situational problem with similar problems solved previously • Recognizes similarities between the situation and the different situational problems • Switches from one type of representation to another and formulates conjectures

Works out a mathematical solution

Uses appropriate strategies based on networks of concepts and processes • Describes the expected result by taking into account the type of information given in the problem • Estimates the order of magnitude of the result, if necessary • Organizes the information • Compares his/her work with the information given in the problem and the tasks to be performed

Shares information related to the solution

Provides a comprehensible and structured oral or written explanation of his/her solution • Takes into account the context, the elements of mathematical language and his/her audience

Validates the solution

Compares his/her result with the expected result • Rectifies his/her solution, if necessary • Assesses the appropriateness and effectiveness of the strategies used by comparing his/her own solution with those of his/her classmates and teacher or with those from other sources • Justifies the steps in his/her procedure

Learning Targets

By the end of the program, students solve situational problems in a variety of everyday or work-related contexts. These more or less complex situations include several steps and address either purely mathematical questions or practical issues (e.g. balancing a budget). Students can use various strategies for representing a situational problem and developing and validating a solution.

As needed, they explore different tentative solutions and use networks of concepts and processes related to one or more branches of mathematics. They present a well-structured solution including a process and a result, and can justify and explain its steps using mathematical language.

Evaluation Criteria

– Oral or written explanation showing that he/she understands the situational problem
– Correct application of mathematical knowledge appropriate to the situational problem
– Development of a solution (i.e. a procedure and a final answer) appropriate to the situational problem

COMPETENCY 2 Uses mathematical reasoning

Mathematical reasoning may be regarded rather schematically as the exercise of two faculties, which we may call intuition and ingenuity.
Alan Turing

Focus of the Competency

Using mathematical reasoning involves making conjectures and criticizing, justifying or refuting a proposition by applying an organized body of mathematical knowledge. This competency will be especially helpful to students in acquiring various skills: observing methodically, inferring and seeking connections between things and events and adjusting their conclusions based on a certain logic, consistency and sequence of events.

The learning and evaluation situations used to develop this competency require that students construct and explain their mathematical reasoning by critiquing a conjecture, whether it is their own or someone else's. To construct their mathematical reasoning and organize their thoughts, they must integrate a functional body of knowledge and skills, along with the relationships between them. Depending on the case, their reasoning may be analogical, inductive or deductive. Analogical reasoning comes to the fore when the teacher helps students perceive and make use of similarities in the purposes of different branches of mathematics. Inductive reasoning is used to identify rules or laws based on observation. Deductive reasoning will help students draw a conclusion based on conjecture and accepted statements.

Students learn to apply these concepts and processes to everyday and work-related situations.

To develop this competency, students must actively participate in activities involving exploration, manipulation, reflection, construction or simulation. They should engage in discussions in which they can use networks of concepts and mathematical processes to justify their choices, compare results, explore activities involving chance and use statistics. They should be encouraged to apply these concepts and processes to everyday and work-related situations, which will oblige them to make use of their sense of observation, intuition, creative thinking, and manual and intellectual skills, as well as their ability to listen to others and express themselves. Given the challenges involved in the development of this competency, constant support by the teacher will be necessary. In previous years, students constructed networks of concepts and mathematical processes by observing various patterns, making connections between numbers and operations, identifying geometric relationships, exploring activities involving chance and interpreting statistical data. They should now be able to mobilize these networks, apply them to the proposed situations and use them to justify actions and statements.

In Prework Training, students continue to learn to construct and use networks of concepts and processes. In order to be able to use mathematical reasoning, students must understand the concepts and processes associated with each branch of mathematics.

The following examples illustrate the contribution of each branch of mathematics to the development of the competency.

- In arithmetic, as in all branches of mathematics, students use their number and operation sense to construct and apply their networks of concepts and processes. They use numbers in different types of notation to interpret data or conclusions in a given context. They also use proportional reasoning when they observe that quantities and sizes are related in a given ratio. They use this type of reasoning to calculate quotients, rates (e.g. slope, speed, flow) and index numbers, to perform operations on or compare elements of number sequences, to convert units or to apply a percentage to a value. They also use it to construct and interpret tables, as well as to analyze statistics and probabilities, and construct and interpret planes and figures.

Given the challenges involved in the development of this competency, constant support by the teacher will be necessary.

- In probability, students learn to incorporate uncertainty into their reasoning by considering all the possibilities and including chance as a parameter. They can verify conjectures through experiments, simulations and the statistical analysis of the data they have collected. In statistics, students plan ways of collecting data, conduct surveys and apply reasoning to the data they have collected. They can recognize and differentiate between the qualitative and quantitative aspects of the data. They use different types of reasoning to prepare a questionnaire and process the data they have collected, which involves organizing the data, choosing the most appropriate way of displaying it, interpreting it and formulating conclusions. The students exercise critical judgment when they evaluate the suitability of the quantitative and graphic methods used to process the data.
- In geometry, students use reasoning when they learn to recognize the characteristics of common figures, identify their properties and perform operations. They also use reasoning to construct figures, compare or calculate unknown measurements, in particular using relations. They infer properties or missing measurements in different contexts using definitions and statements.

The competency *Uses mathematical reasoning* has three key features: *Defines the conditions inherent in a mathematical situation*; *Selects and applies networks of mathematical concepts and processes*; and *Justifies actions or conjectures using mathematical concepts and processes*.

Key Features of Competency 2

Uses mathematical reasoning

Defines the conditions inherent in a mathematical situation

Identifies elements related to the situation • Recognizes patterns or constraints in various situations • Identifies relationships between mathematical concepts and processes in different situations • Formulates a likely or plausible opinion • Assimilates or states conjectures adapted to the situation, if necessary • Plans the application of one or more processes

Selects and applies networks of mathematical concepts and processes

Uses different types of representations • Constructs concepts and processes • Identifies the elements of mathematical language pertaining to these concepts and processes • Networks different concepts associated with the situation • Assesses the relevance of the selected concepts and processes, if necessary

Justifies actions or conjectures using mathematical concepts and processes

Uses counterexamples to clarify, adjust or refute conjectures, if necessary • Formats the results of the process • Uses the appropriate mathematical language • Reflects on his/her process and revises it as needed

Evaluation Criteria

– Oral or written demonstration of his/her understanding of the situation
– Correct use of the concepts and processes selected
– Oral or written justification of an action or series of actions appropriate to the situation

Learning Targets

By the end of the program, students apply concepts and processes appropriate to the situation in order to confirm or refute conjectures or statements in everyday or work-related situations (e.g. determining whether it is possible to rent an apartment on a given salary). They demonstrate an understanding of concepts and processes associated with one or more branches of mathematics. They use mathematical reasoning and structure their process using strategies such as observing methodically or looking for connections between facts and events in order to draw conclusions.

Mathematics is not merely a simple language that expresses observations.
More than any other, the language of mathematics is inseparable from the process of thought itself.
Gaston Bachelard

Focus of the Competency

In a world where communication combines everyday and mathematical language[4] (terms, symbols and notation), it is important that students develop an understanding of the specific elements of mathematical language and learn to use them as needed. Graphic representations in the media, survey results, nutritional equivalence charts and directions for using household appliances all require the use of mathematical language in everyday life.

The competency *Communicates by using mathematical language* is required in many jobs, for example to read and produce invoices, read and interpret data on various dials and graduated scales, or report on the number of clients served. It requires accuracy and rigour and is demonstrated in particular in clear, concise messages. To develop the competency, students must practise interpreting and producing messages of varying complexity in situations that require them to use mathematical types of representation, concepts or processes.

Students discover the usefulness of mathematical language in a variety of everyday activities.

In previous years, students interpreted, produced and conveyed oral or written messages, using different types of representation. They refined their choice of mathematical terms and symbols. They compared information from various sources. In discussions with classmates, they analyzed different points of view and adjusted their message if necessary.

The following examples illustrate the contribution of each branch of mathematics to the development of the competency.

- In arithmetic, students communicate when they produce and interpret numerical expressions. They represent relationships between the elements of a situation using everyday and symbolic language. They express and justify their point of view and choices when they explain the effect of modifying certain data. Lastly, their communications rely on their number and operation sense and they choose mathematical elements, units and the appropriate notation for the message they wish to convey.
- In statistics and probability, students communicate when they enumerate (table, tree diagram) and estimate the probability of a simple event. When they organize, represent, analyze and interpret data, they highlight certain information, selecting the relevant types of representation. They represent the situation using diagrams or graphs, develop a questionnaire if necessary and present their results. They formulate arguments to justify their conclusions.
- In geometry, students communicate when they construct geometric figures and describe their properties. They use definitions to produce a clear and coherent message. When they are looking for unknown measurements, they communicate using metric relationships and the appropriate units of measurement and by producing or interpreting formulas.

The competency *Communicates by using mathematical language* has three key features: *Analyzes a situation involving mathematical communication*; *Interprets or conveys mathematical messages*; and *Produces mathematical messages*.

4. Appendix A contains examples of elements of mathematical communication.

Key Features of Competency 3

Communicates by using mathematical language

Analyzes a situation involving mathematical communication

Identifies the purpose of the message • Distinguishes between the everyday and mathematical meaning of various terms • Consults different sources of information when necessary • Organizes his/her ideas and establishes a communication plan

Interprets or conveys mathematical messages

Expresses his/her ideas using mathematical language, taking into account its rules and conventions as well as the context • Validates a message to make it more comprehensible, if necessary • Summarizes information • Has discussions based on mathematical messages

Produces mathematical messages

Chooses the elements of mathematical language that suit the context and the message • Associates images, objects or concepts with the mathematical terms and symbols, depending on the context • Selects types of representations that suit the message and the audience

Evaluation Criteria

– Correct interpretation of a mathematical message
– Production of an oral or written message appropriate to the situation

Learning Targets

By the end of the program, students know how to interpret and analyze mathematical messages, and to critique and improve them based on the requirements of the situation. They interpret, produce and transmit oral and written messages using everyday and mathematical language. The messages they produce are clear, coherent and adapted to the situation (e.g. giving directions). They use different types of representation to demonstrate their understanding of the elements of a message or to produce a message.

Program Content

Competency development in the Mathematics program is based on a body of resources, knowledge and skills made up of concepts, processes and strategies. The elements of the program content are divided into three branches: arithmetic, statistics and probability, and geometry. In the Work-Oriented Training Path, teachers choose the elements to be studied based on students' abilities, needs and interests.

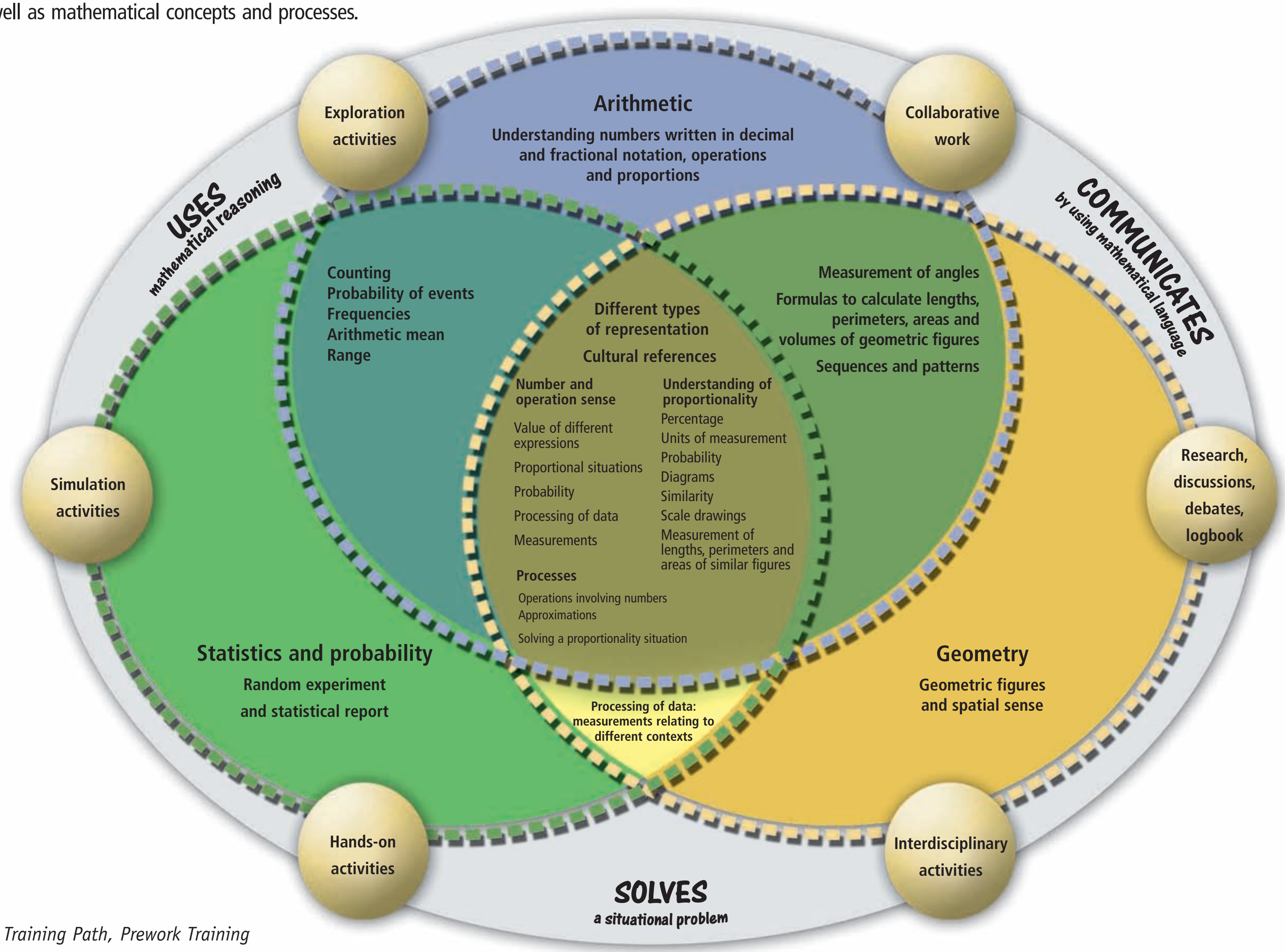
Students must construct concepts and processes and apply them in a variety of contexts. Since the progression of learning in mathematics is based on prerequisites, the program content must be addressed in a spirit of continuity, i.e. taking students' learning into account. The elements of content are based on the assumption that there are links between the different branches of mathematics and between mathematics and other subjects. The properties of objects in a given branch contribute to students' understanding of the objects in other branches. Since the branches are interdependent and provide mutual enrichment, the elements are acquired in synergy.

In addition, teachers are faced with two major concerns: first, ensuring that students learn the meaning of the concepts and processes before memorizing them and developing the automatic responses that will enable them to link these concepts and processes in different mathematical operations; second, encouraging them to use mathematical language as a communication tool inherent in the subject and essential for understanding, interpreting and discussing certain phenomena. Simply having students master the vocabulary and automatic responses, to the detriment of all else, will never enable them to confidently solve real mathematical problems or situations.

The content for each branch of mathematics is presented in a table that includes concepts, processes and possible applications. Other tables give examples of strategies used in all three branches: cognitive and metacognitive strategies, affective strategies and resource management strategies. The four appendixes contain examples of elements of mathematical communication (Appendix A), types of representation (Appendix B), examples of different meanings of mathematical operations to be carried out in learning and evaluation situations (Appendix C) and different ways of solving proportionality situations (Appendix D).

Intradisciplinary Links

This diagram illustrates the various intradisciplinary links that must be considered in constructing mathematical knowledge. These links involve the different branches of mathematics, as well as mathematical concepts and processes.



Arithmetic

Number sense and operation sense

In previous cycles, students developed their sense of numbers and operations involving natural numbers, fractions and decimals. They identified the properties of operations as well as the relationships between them. They now know how to follow the order of operations in simple sequences of operations. They were introduced to the concept of integers. To a certain extent, they are able to perform operations involving natural numbers, decimals and, with considerable help from the teacher, fractions, using objects and diagrams.

In Prework Training, students must develop and master their learning. If necessary, they visualize operations using concrete materials such as strips of paper and blocks. They should also be able to give meaning to numerical operations by using them regularly to do mental or written computation or computations with a calculator. An understanding of operations is also acquired in a variety of contexts. For example, addition and subtraction can be used in situations that involve uniting, comparing or transforming. Multiplication can be used in cases involving comparison, combination or rectangular arrangement, and division, in situations that involve sharing or capacity. Appendix C provides examples of addition, subtraction, multiplication and division problems.

To develop their sense of numbers and operations involving numbers, students construct and assimilate the following concepts and processes:

<table>
<tr><th colspan="3">Number Sense and Operation Sense</th></tr>
<tr><th>Concepts</th><th>Processes</th><th>Possible Applications</th></tr>
<tr>
<td>Number sense with regard to decimal and fractional notation and operation sense with regard to integers
– Reading, writing, various representations, patterns, properties (e.g. even number, square, prime number, composite number)
– Decimal, fractional and exponential (integral exponent); percentage, square root
– Properties of divisibility (by 2, 3, 4, 5, 10, etc. depending on the context and needs)
– Rules of signs for addition and subtraction
– Equality relation: meaning of =, properties and rules for transforming numerical equalities (balancing equalities)
– Inverse operations: addition and subtraction, multiplication and division, square and square root
– Properties of operations:
• Commutative and associative properties
• Distributive property of multiplication over addition or subtraction and factoring out the common factor
– Order of operations and the use of no more than one level of parentheses
Notes:
Learning activities must focus on helping students develop their number sense and operation sense.
Students should still be encouraged to use the proper terms learned in previous cycles (natural numbers, integers, decimals).

Knowledge of the properties of operations helps students think of equivalent ways of writing numbers and operations, which simplifies computations and can eliminate dependence on a calculator.
Knowledge of the order of operations helps students use technology (e.g. a calculator).</td>
<td>Different ways of writing and representing numbers
– Estimating the order of magnitude
– Comparing
– Using a variety of representations (e.g. numerical, graphic)
– Recognizing and using equivalent ways of writing numbers:
• Decomposition of numbers (e.g. additive, multiplicative)
• Equivalent fractions
• Simplification and reduction
– Switching from one way of writing numbers to another or from one type of representation to another (from 0.5 to ½ or 50%)
– Transforming arithmetical equalities
– Locating numbers on a number line
Note:
Positive or negative numbers written in decimal or fractional notation are used on a number line or in a Cartesian plane. Students should work with positive numbers when switching from one way of writing numbers to another.
Operations involving numbers written in decimal and fractional notation
– Estimating and rounding numbers in different situations
– Looking for equivalent expressions
– Approximating the result of an operation
– Simplifying the terms of an operation</td>
<td>– Reading a thermometer
– Balancing a budget: housing, food, recreation, etc.
– Calculating a temperature increase starting at minus one degree (sign rules)
– Comparing bank fees at different institutions
– Estimating the cost of purchasing a car versus paying for public transportation
– Calculating the interest on loans in order to make a decision: to pay cash or make monthly installments
– Calculating daily calorie intake and use in order to maintain a healthy weight
– Comparing the cost and benefits of buying prepared meals or cooking them themselves
– Calculating their weekly salary based on the number of regular and overtime hours worked
– Calculating the savings involved in a percentage discount on an item purchased</td>
</tr>
</table>

Number Sense and Operation Sense		
Concepts	**Processes**	**Possible Applications**
	– Mental computation: • The four operations with positive numbers written in decimal notation • Continued construction and integration of their memorized repertoire – Written computation: • The four operations involving positive numbers that are easy to work with (including large numbers) and sequences of simple operations performed in the proper order (numbers written in decimal notation) • Addition and subtraction using numbers written in decimal notation (positive and negative numbers) – Use of a calculator: the four operations and sequences of operations performed in the proper order **Notes:** Students use technological tools for operations in which the divisors or multipliers have more than two digits. Depending on students' specific needs, the use of a calculator may be permitted at all times. For written computation, the understanding and mastery of processes is more important than the ability to do complex computations.	

Certain conjectures can be used to help students develop number and operation sense, for example:

– The product of two strictly positive numbers is greater than or equal to each of the two numbers.

– If an integer ends with the digit 2, then it is an even number.

Understanding proportionality

Developing an understanding of proportionality means honing one's ability to compare, relate and estimate a ratio. The concept of proportionality is everywhere in daily life: calculating interest rates, quantities of ingredients, percentages, etc. To be able to translate a situation using a proportion, students must be able to recognize that the situation involves proportionality. An understanding of proportions can be developed when students interpret ratios or rates in various situations, compare them qualitatively or quantitatively (e.g. "*a* is darker than *b*," "*c* is *x* times more concentrated than *d*") and describe the effect of changing a term, a ratio or a rate. In Prework Training, the development of proportional reasoning is important and its applications are numerous. Teachers can help students develop this skill by placing them in situations which, for example, oblige them to use percentages (calculating a certain percentage of a number and the value corresponding to 100 per cent) in situations related to consumption and statistics, or to make scale models and construct circle graphs in working with graphs.

Appendix D contains an example of a proportionality situation involving different problem-solving strategies.

Understanding Proportionality		
Concepts	**Processes**	**Possible Applications**
Understanding proportionality – Ratio and rate • Ratios and equivalent rates • Unit rate – Proportion • Equality of ratios and rates • Ratio and proportionality coefficient **Note:** The program does not address inverse variation.	***Working with a proportionality situation*** – Comparing ratios and rates – Recognizing a proportionality situation by referring to the context, a table of values or a graph – Solving a proportionality situation – Finding ordered pairs in a Cartesian plane (abscissa and ordinate of a point)	– Comparing the percentage of their weekly allowance devoted to lunches with that devoted to recreational activities – Calculating the cost price per muffin for a dozen muffins – Comparing unemployment rates in different regions and different trades and occupations – Evaluating gas savings as a result of driving at lower speeds – Expressing a weekly salary as an hourly wage – Calculating the unit price per tile based on the overall cost price of a ceramic floor – Measuring the proportion of water needed to dilute a household cleaner – Adapting a cocktail recipe for two to serve six people

Statistics and Probability

Probability

In previous cycles, students conducted experiments related to the concept of chance. They made qualitative predictions about outcomes by becoming familiar with the following concepts: the certainty, possibility and impossibility of an event and the probability that an event will occur (more probable, equally probable, less probable). In Prework Training, students develop their critical thinking skills with respect to probability for the purpose of making decisions. Experiments, real-life situations, games, diagrams, graphs and sketches make it easier to understand random phenomena. Repeating an experiment makes it possible to assimilate certain concepts related to phenomena involving chance.

Understanding Data From Random Experiments		
Concepts	**Processes**	**Possible Applications**
Random experiment – Random experiment • Possible results	***Processing data from random experiments*** – Conducting random experiments – Predicting a result (certain, possible or impossible) – Enumerating possibilities using a tree diagram or table – Calculating the probability of a simple event (more probable, equally probable, less probable)	– Enumerating possible results in a variety of situations: • Tossing a coin, a die, two dice • Participating in a drawing • Drawing a card from a deck – Making decisions related to the probability of an event in the previously mentioned situations

Statistics

In previous cycles, students conducted surveys (they learned how to formulate questions, gather data and organize it using tables). They also interpreted and displayed data using bar graphs, pictographs and broken-line graphs. They interpreted circle graphs and calculated the arithmetic mean of a distribution. In Prework Training, statistics helps students develop their critical judgment. To be able to draw conclusions or make informed decisions based on the data or empirical results of a survey, students must know all the steps involved in conducting it. They can learn this by applying each of these steps to a problem they have isolated and that relates to different situations. They devise a short questionnaire and choose a representative sample of the population being studied. They gather data, organize them using a table, display them in a graph and derive information that will allow them to interpret the results. They choose the graphs that provide an appropriate illustration of the situation.

Meaning of Statistical Data		
Concepts	**Processes**	**Possible Applications**
Statistical report – Population, sample • Survey, census • Representative sample • Sampling methods: simple random, systematic • Sources of bias – Data • Qualitative variable • Quantitative variable – Table: characters variables, frequencies of variables, – Reading graphs: bar graphs, broken-line graphs, circle graphs – Arithmetic mean – Range	***Processing data from statistical reports*** – Conducting a survey or a census • Determining the population or the sample • Gathering data – Organizing and choosing certain tools to present data • Constructing tables • Constructing graphs: bar graphs, broken-line graphs, circle graphs – Highlighting some of the information that can be derived from a table or a graph (e.g. minimum value, maximum value, range, mean)	– Discussing drug and alcohol use based on statistics – Analyzing statistics on water and electrical consumption and their environmental impact: for example, taking a bath versus taking a shower – Finding information about the effectiveness of various means of contraception – Consulting statistics on available jobs related to the students' interests – Using statistics to find the basic salary[5] for different jobs – Conducting a survey on the recreational preferences of adolescents

5. The Statistics Canada Web site may be useful in this regard: http://cansim2.statcan.ca/cgi-win/cnsmcgi.pgm?Lang=E&SP_Action=Sub&SP_ID=1803.

Geometry

In previous cycles, students located numbers on a number line and in a Cartesian plane. They constructed and compared different solids (prisms, pyramids, spheres, cylinders and cones), focusing on prisms and pyramids. They recognized the nets of convex polyhedrons and described and classified quadrilaterals and triangles. They became familiar with the features of a circle (radius, diameter, circumference, central angle). They observed and produced frieze patterns and tessellations by means of reflections and translations. Lastly, they estimated and determined different measurements: lengths, angles, surface areas, volumes, capacities, masses, time and temperature.

In Prework Training, students are required to use their geometric thinking skills and spatial sense in their everyday activities, in different contexts relating to mathematics or other subject areas, or to meet various needs (e.g. getting their bearings, reading a map, evaluating a distance, playing computer games). To develop their spatial sense in three dimensions, which requires a certain amount of time, the students draw solids freehand. They identify solids by means of their nets or their representations in the plane. They recognize plane figures obtained by the intersection of a solid with a plane.

Certain measuring instruments have remained virtually unchanged through the ages, while others have been perfected; the students discover them as well as the use of different units of measurement. They are also introduced to the imperial and metric systems in certain spheres of human activity. "Paper-and-pencil" constructions and the use of the appropriate software are two ways of helping students develop their sense of measurement and compare perimeters and areas in different contexts. In order to determine an unknown measurement and justify the steps in their procedure, the students will rely on definitions and properties rather than on measurement. They apply concepts and processes related to arithmetic and proportions.[6]

6. The Web site of the Service national du RECIT Mathématique, Science et Technologie contains useful geometrical tools: http://recitmst.qc.ca/AppsMath/.

Geometric Figures and Spatial Sense		
Concepts	**Processes**	**Possible Applications**
Geometric figures[7] and spatial sense – Figures planes • Triangles, quadrilaterals and regular convex polygons - Segments and lines - Base, height • Circles and discs - Radius, diameter - Central angle • Measurement - Angle in degrees - Length - Perimeter, circumference - Area - Volume - Choice of unit of measurement for lengths or areas - Relationships between units of length in the imperial system - Relationships between units of area in the metric system (SI) • Angles - Complementary, supplementary • Solids - Right prisms, right pyramids and right cylinders - Possible nets of a solid - Decomposable solids • Congruent and similar figures	– Constructing geometric figures – Finding unknown measurements • Lengths - Perimeter of a plane figure - Circumference of a circle - Unknown measurement of a segment in a plane figure • Areas - Area of polygons that can be split into triangles and quadrilaterals - Area of discs - Area of figures that can be split into discs, triangles or quadrilaterals - Area of right prisms, right cylinders and right pyramids - Area of solids that can be split into right prisms, right cylinders or right pyramids • Volume - Volume of right prisms and right cylinders • Angles - Unknown measurement in different situations **Note:** The processes related to geometric constructions are used to build concepts that can be applied in different situations, and for the development of the students' spatial sense. These constructions can be done using appropriate geometry sets or software.	– Managing their time by estimating the amount of time needed to perform certain tasks – Planning the layout of a room by estimating the volume of the furniture – Converting pounds to kilograms, miles to kilometres, gallons to litres and hours to minutes – Giving directions – Using the measurement system used in some areas, often the imperial system, for example: cutting 5¼ inches off the length of a beam or hammering in a row of nails 3½ inches apart – Establishing a delivery schedule to clients' satisfaction – Estimating the space needed to display a product on store shelves – Purchasing paint to repaint a room – Calculating the quantity of sand or cement needed for a given job

Some geometric statements can be given special attention, depending on the job at hand. For example, when cutting tiles, the following statements could prove useful:
– The sum of the measures of the interior angles of a triangle is 180°.
– The ratio of the circumference of a circle to its diameter is a constant known as pi (π).

7. In a geometric space of a given dimension (0, 1, 2 or 3), a geometric figure is a set of points representing a geometric object such as a point, line, curve, polygon or polyhedron.

Strategies

Cognitive and metacognitive strategies are involved in the development and exercise of the three mathematics competencies; they are integrated into the learning process. Some of them can be given special attention depending on the situation and the targeted objective. Since the students must construct a personal repertoire of strategies, they should be encouraged to develop their autonomy in this respect and learn how to use such strategies in different contexts.

The students learn the importance of making connections between the development of mathematics competencies and the concrete application of the strategies they have used in the past to overcome difficulties. They will learn to apply their competencies in different situations to solve problems in mathematics, as well as in everyday life. For some students, learning mathematics means that they will need to meet particular challenges. Thus, the use of affective and motivational strategies might prove essential.

Cognitive and Metacognitive Strategies	
Strategies	**Reflection**
Planning	– Have I determined what needs to be done? – Did I apply my prior knowledge of the subject? – Did I identify the relevant information? – Did I need to break the problem down? – Did I estimate the amount of time required?
Understanding	– What terms appear to have a different meaning in mathematical language than they do in everyday language? – Did I need to find a counterexample to prove that my statement was false? – Was all of the information provided in the situation relevant?
Organization	– Did I group together, enumerate, classify and compare data or use diagrams? – Did I choose the appropriate concepts? – Are the important parts of my procedure well represented
Development	– Did I represent the situation in my head or in writing? – Did I refer to a similar problem I had already solved? – What information did I identify based on the information provided? – Did I identify the important parts of the question? – Did I write down comments and questions in my own words?

Cognitive and Metacognitive Strategies	
Strategies	**Reflection**
Regulation	– Did I use the correct procedure and can I explain it? – Can I verify my solution using reasoning and an example or counterexamples? – What did I learn? How did I learn it? – Did I choose the correct strategy and take the time needed to fully understand the problem? – What are my strengths and difficulties? – Did I adjust my method to the task at hand? – What is the expected outcome? – What is the reason for the difference between the expected outcome and the actual outcome? – What strategies did my classmates use or did the teacher suggest that I could add to my repertoire? – Could I use this procedure in other situations?
Generalization	– Did I find similarities and differences in the examples? – Did I find models that I can use again later? – Are observations made in a particular case applicable to other situations? – Are the statements made or conclusions drawn always true? – Did I identify examples and counterexamples?
Repetition	– What methods did I use: repeating several times (in my head, in a whisper, out loud), highlighting, underlining, circling, copying, making lists of terms and symbols, etc.? – Could I do the problem again without help? – What characteristics of situations led me to repeat the same strategy?
Automation of a process	– Did I find a model solution and make a list of steps? – Have I practised enough to be able to apply the procedure automatically? – Can I effectively use the concepts I learned? – Did I compare my procedure with that of others?
Communication	– Did I mobilize different types of representation? – Did I show enough of my work? – Did I try out different ways of conveying my mathematical message? – Did I use an effective means of conveying my message? – Would other means have been just as effective, more effective or less effective?

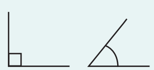
Other Strategies	
	Reflection
Affective strategies	– What did I like about this situation? – Am I satisfied with my work? – What was I particularly good at in this situation? – What means did I use to overcome difficulties and which ones helped me the most to: • reduce anxiety? • maintain my level of concentration? • control my emotions? • stay motivated? – Did I take risks? – Can I recognize my achievements?
Resource management strategies	– Who can I ask for help and when? – Do I accept the help offered? – Did I consult documents? – Did I consult my tool kit (e.g. references, glossary, posters)? – Did the hands-on material help me solve the problem? – Did I correctly estimate the amount of time needed to do the activity? – Did I successfully plan my work periods: shorter, more frequent periods; objectives for each period; etc.? – Did I use the appropriate means to maintain my level of concentration: appropriate environment, materials available?

APPENDIX A – ELEMENTS OF MATHEMATICAL COMMUNICATION

Types of sentences

- Sentences containing only words
  - Example: True or false? If a diamond has four right angles, it is a square.
- Sentences containing words and mathematical symbols
  - Example: What is the value of the expression (7 + 6) – 3 X 4?
- Sentences containing only mathematical symbols
  - Example: 3 x 4 = 12

Types of symbols

- Symbols used to name objects
  - Examples: $8, \frac{3}{5}, \angle$
- Symbols used in operations
  - Examples: $+, -, \times, \div, \sqrt{\ }$
- Symbols used in relations
  - Examples: $>, <, =, \neq, \perp$
  - Graphic symbols
  - Examples: 

Meaning of symbols

- The order and position of symbols affects their meaning.

  Examples:

  34 and 43

  $\frac{3}{5}$ and $\frac{5}{3}$

  1,234 and 12,34 and 123,4

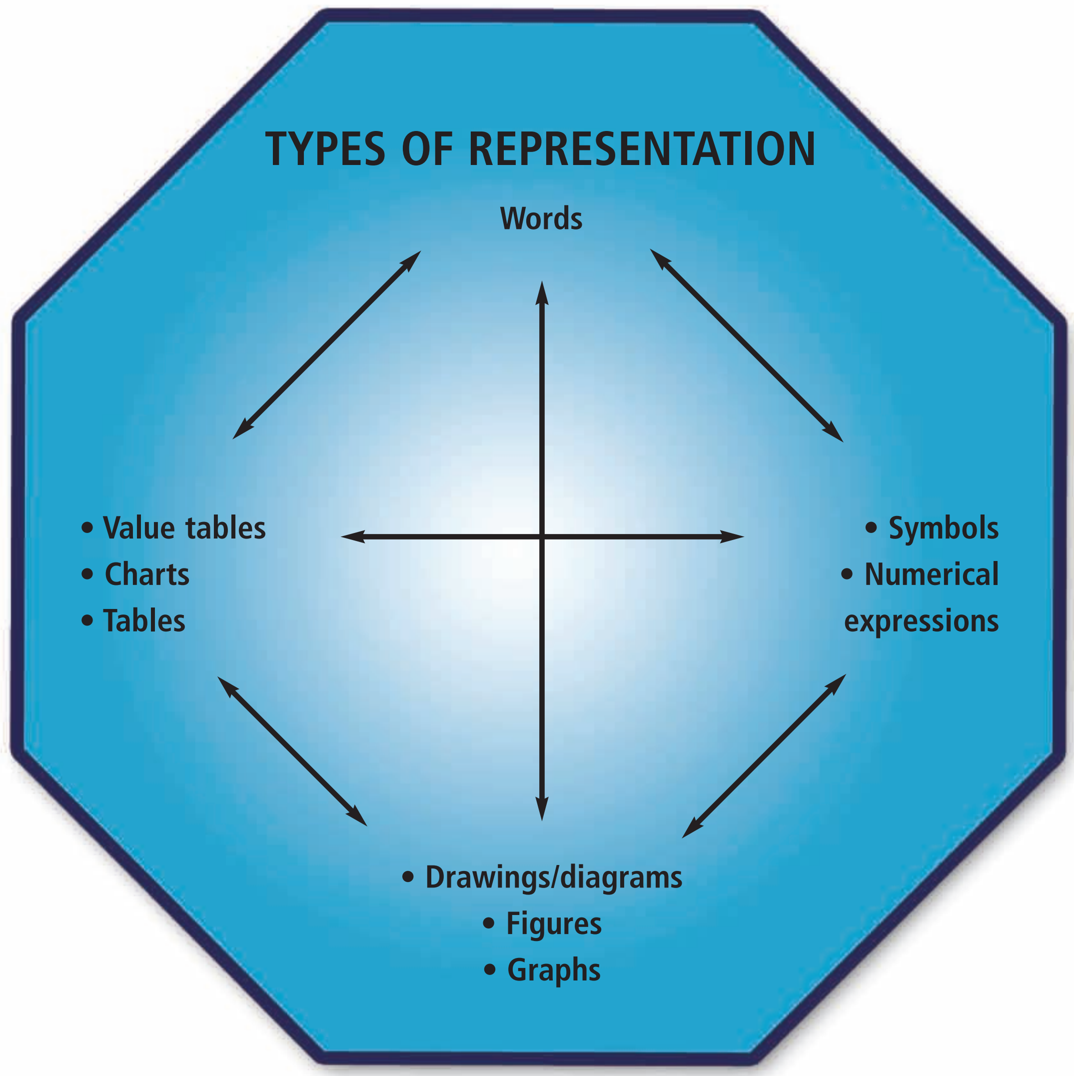
  3^2 and 2^3

Terms and their meaning

- Terms with the same meaning in mathematics and in everyday language
  - Examples: length, line, area
- Terms with a different meaning in mathematics than in everyday language
  - Examples: product, factor, volume
- Terms with a more precise meaning in mathematics than in everyday language
  - Examples: division, average, reflection

Reading of symbols and expressions

- Different expressions for reading
  = . . . equals . . .
  ≥ . . . greater than or equal to . . .
- Different expressions for reading 12 – 5
  twelve minus five; twelve subtract five;
  five less than twelve; take five away from twelve;
  the difference between twelve and five


TYPES OF REPRESENTATION
Words
• Value tables
• Charts
• Tables
• Symbols
• Numerical expressions
• Drawings/diagrams
• Figures
• Graphs

APPENDIX C – EXAMPLES OF DIFFERENT MEANINGS OF THE FOUR OPERATIONS, ILLUSTRATING THE NEED TO VARY SITUATIONS

Addition and subtraction:

- **Uniting**

  Max has two red notebooks and three blue notebooks. How many notebooks does he have in all?

  Max has five notebooks in all. Two are red, the others are blue. How many blue notebooks does he have?

- **Comparing**

  Max has five dollars and Maude has ten. How many dollars more does Maude have than Max? Or how many dollars less does Max have than Maude?

  Max has five dollars, Maude has five dollars more than he does. How many dollars does Maude have?

  Maude has ten dollars, Max has five dollars less than she does. How many dollars does Max have?

- **Transforming**

  Max has five posters. Maude gives him five more. How many posters does Max have now?

  Max has five posters. Maude gives him some more. He now has ten posters. How many posters did Maude give him?

  Maude has some posters. She gives five to Max. She now has five posters. How many posters did she have before she gave five to Max?

Multiplication:

- **Comparing**

  Max has fifteen CDs. Maude has three times as many. How many CDs does Maude have?

- **Combining**

  Max has four pairs of pants and six t-shirts. How many different combinations can he wear?

- **Rectangular arrangement**

  The library has seven shelves, each containing ten books. How many books are there in the library?

Division:

- **Sharing**

  Maude has twelve CDs that she wants to divide evenly among her three friends. How many CDs will each friend receive?

- **Capacity**

  Max has twelve CDs and wants to give four to each of his friends. How many of his friends will get CDs?

APPENDIX D – EXAMPLE OF DIFFERENT WAYS OF SOLVING A PROPORTIONALITY SITUATION

The students solve a proportionality situation using different multiplicative strategies that they will have developed (e.g. unit-rate method, factor of change, ratio or proportionality coefficient, additive procedure or mixed procedure).

A minimum of three ordered pairs is required to analyze a proportionality situation using a table of values.

Table of Values

Quantity of product A	2	4	6	10
Quantity of product B	6	12	18	?

Different Ways of Solving the Proportionality Situation

Unit-rate method	If for 1 unit of product A, we have 3 units of product B (12 ÷ 4); then for 10 units of product A, we will have (10 X 3) units of product B.
Factor of change	The factor making it possible for 4 to be increased to 10 is 2.5; we apply this factor to 12.
Proportionality coefficient	The factor making it possible for 4 to be increased to 12 is 3; we apply this factor to 10.
Additive procedure	Since 4:12 = 6:18, then: $\frac{4}{12}=\frac{6}{18}=\frac{4+6}{12+18}=\frac{10}{30}$

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